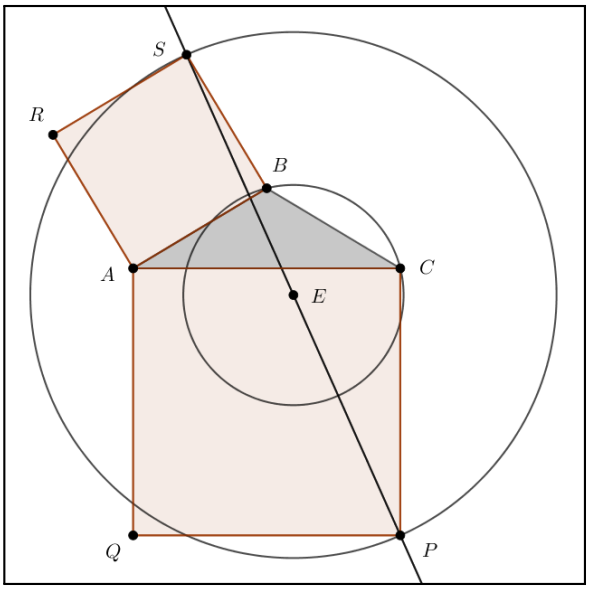
Squares on Two Sides of a Triangle

#8. $ACPQ$ and $BARS$ are squares on the sides AC and BA of a triangle ABC. If B and C remain fixed, whilst A varies, prove that the line PS always passes through a fixed point (Pedoe 1988, Problem 14.3, p. 66).

Proof. Let τ_B be the counter-clockwise rotation around point B taking S to A, namely, a rotation by $+90°$ around B. Likewise, τ_C is the counter-clockwise rotation around point C taking A to P, also a counter-clockwise rotation by $+90°$, but around C. The composition of these rotations $\tau_C \circ \tau_B$ (understood to to apply τ_B first) is then itself a rotation taking S to P. The angle for rotation $\tau_C \circ \tau_B$ is $+90° + +90° = +180°$, the sum of the component rotation angles. That is, rotation $\tau_C \circ \tau_B$ rotates counter-clockwise around some point by $180°$. A rotation in the plane takes exactly one point to itself, namely the center of rotation; it's clear that this point must be the midpoint of segment SP, denoted by E in the diagram.



Nothing in the foregoing depends on point A, consequently point E will be the same no matter how point A is chosen. **QED.**

– Mike Bertrand
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