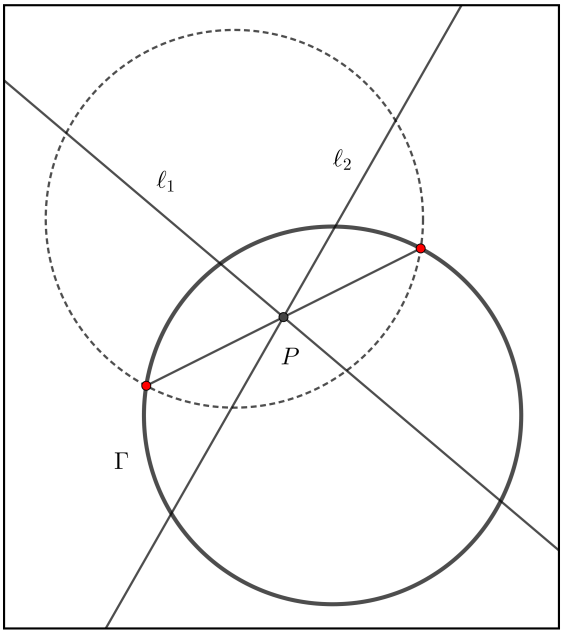
Circle with Two Lines Intersecting Within

#4. Given a circle and two straight lines intersecting inside the circle, construct a chord which is bisected by the point of intersection of the two lines.



Construction. Two distinct lines ℓ_1 and ℓ_2 intersect at point P inside a circle, outlined in bold here. Consider the transformation reflecting any point in the plane through point P (a dilation from P with scale -1). Such transformations take circles to circles and in this particular case the second circle is the same size as the original one — the dashed circle is the image of the original circle in this diagram. The two circles intersect at more than one point. For P maps to itself. And the point Q halfway between P and the point on the original circle closest to P is also in both circles (the only objection would be if P were the center of the original circle, in which case the solution is trivial). So the two circles intersect at two points, the red points in the diagram. The segment connecting those points is a chord on the original circle, and since the two points are images of each other under the transformation, P bisects the segment connecting them. **QEF**.

A simple continuity argument proves that there is such a solution. To illustrate with this diagram, rotate line ℓ_2 clockwise around P until it coincides with ℓ_1. Looking at the chord produced by ℓ_2, the part northeast of P is shorter that the part southwest of P. The situation has reversed after ℓ_2 has rotated to coincide with ℓ_1, so by continuity there must be a point along the arc where the two parts of the chord are equal (the red point at the upper right!). Nice, but not as nice as the original solution, which actually produces the points leading to a bisected chord.

– Mike Bertrand
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