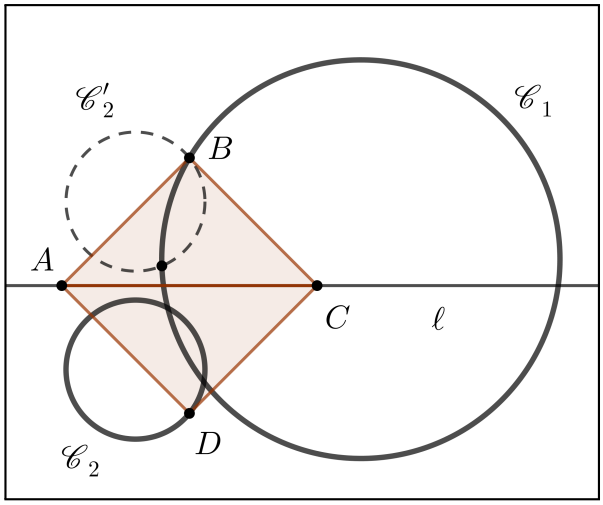
Two Circles, A Line, and a Square

#2. Two circles $\mathscr{C}_1$ and $\mathscr{C}_2$ are given, and a line ℓ. Construct squares $ABCD$ with A and C on ℓ, with B on $\mathscr{C}_1$, and D on $\mathscr{C}_2$ (Pedoe 1970, Problem 41.2, p. 163).

Construction. Consider this diagram, where the circles are as labeled, with the thick solid outlines. Line ℓ is the horizontal line riding just above $\mathscr{C}_2$. The second smaller circle $\mathscr{C}_2'$ with the dashed outline is the reflection of $\mathscr{C}_2$ around line ℓ. Suppose $\mathscr{C}_2'$ intersects $\mathscr{C}_1$ at point B. Construct the square with vertex at B and diagonal AC along line ℓ. By the symmetry of the arrangement, D is on $\mathscr{C}_2$, and $\square ABCD$ is a solution. Note that in general $\mathscr{C}_2'$ intersects $\mathscr{C}_1$ at a second point B' and another square solving the problem can constructed in the same way with vertex B' — in this case leading to a tiny square not shown. If $\mathscr{C}_1$ and $\mathscr{C}_2$ are internally or externally tangent, the two solutions collapse into one.



It is also necessary that $\mathscr{C}_1$ intersect $\mathscr{C}_2'$ for a solution to exist. For suppose such a square $ABCD$ exists. Then point D on $\mathscr{C}_2$ must be mirrored in point B, which has to be on both $\mathscr{C}_2'$ and $\mathscr{C}_1$. **QEF.**

– Mike Bertrand
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