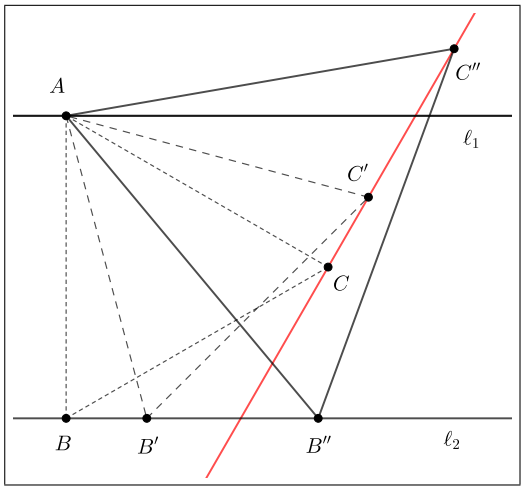
Equilateral Triangle on Three Parallel Lines

#1. Given three distinct parallel straight lines in the plane, construct an equilateral triangle with a vertex on each line.



Construction. The lines can be assumed to be horizontal; denote the top one ℓ_1 and the bottom one ℓ_2 (the third line, unshown, lies between them). Pick a point A on ℓ_1 and drop the perpendicular down to point B on ℓ_2. Construct an equilateral triangle on segment AB, with third vertex C to the right. Pick any point B' on ℓ_2 to the right of B and construct the equilateral triangle $\triangle AB'C'$, also to the right. In the same way, construct equilateral $\triangle AB''C''$, where B'' lies on ℓ_2 to the right of B'. The transformation taking each B to its corresponding C is a counter-clockwise rotation around A by $\pi/3$. Such a transformation takes lines to lines, so the image of ℓ_2 under this transformation is also a line, the red one as pictured. Put another way, all the Cs lie on a straight line, and this line is not parallell to the others.

The solution is now clear. Take the point C^{iv} on the red line intersecting the third parallel line between ℓ_1 and ℓ_2. Since the transformation is $1-1$, it corresponds to a point B^{iv} on ℓ_2 and these two points together with point A are the vertices of an equilateral triangle with one vertex on each of the three parallel lines. **QEF**.

– Mike Bertrand
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