Fast Bezier Curves in Windows

Michael Bertrand

B ezier curves are widely used in computer graphics. Post-Script uses these curves as building blocks, for example; defining even circles in terms of Beziers. Many modern PC and Mac illustration programs implement Bezier curves in a direct way, allowing users to create and interactively edit the curves to create complex images.

These curves possess several properties that have led to their widespread adoption in computer graphics applications:

- Because they are defined in terms of a few points, Bezier curves can be identified with these points in the graphics database.
- Efficient algorithms generate the entire curve from the defining points.
- The defining points intuitively describe the curve.

Four defining points—A Bezier curve is defined by four points called control1, handle1, handle2, and control2 (abbreviated here as ctrl1, hand1, hand2, and ctrl2). The curve begins at ctrl1 and proceeds toward hand1; at the other end, we can think of it as starting from ctrl2 and proceeding toward hand2. In each case, the curve gradually pulls away from the associated handle in its movement toward Here's how to draw Bezier curves quickly enough to rubber-band them on-screen.

the other handle/control. The handles are points of attraction for the curve; the curve starts from ctrl1 toward hand1, but gradually pulls toward hand2 as the attractive force of hand1 diminishes and the attractive force of hand2 increases. The further away hand1 is from ctrl1, the longer the curve will be pulled toward ctrl1 before breaking toward hand2.

Bezier curves take several forms, as shown in the screen shot given in Figure 1. Vectors connecting each control point to its handle are shown as well as the Bezier curve they define. This example is about as complex as four-point Beziers get; more complex Bezier curves require a series of Beziers joined together in a continuous path.

de Casteljau Construction—Beziers must be drawn quickly to be dragged, or rubber-banded, on the screen, since the curves are being drawn and erased constantly with each mouse movement. We also need fast Bezierdrawing routines to render a complex image comprising perhaps hundreds of individual Beziers in a reasonable time.

The de Casteljau algorithm is a fast integer-based method for calculating points along a Bezier curve, given the four original defining points. The calculated points can then be connected by line segments to give the impression of a smooth curve. The de Casteljau algorithm breaks a Bezier curve into two separate pieces, left and right, each of which is itself a Bezier curve.

The key to the efficiency of this algorithm is the astounding simplicity of the math, which involves taking simple averages to calculate de Casteljau construction points: First average the original defining points (the q's shown below are the averages), then average the averages (r's), then take a final average (s0):

```
ctr11 = p0

q0

hand1 = p1 r0

q1 s0

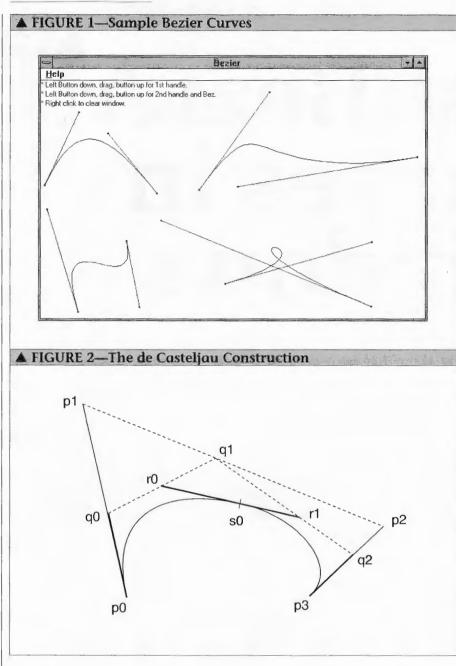
hand2 = p2 r1

q2

ctr12 = p3
```

That is: q0 = (p0 + p1)/2, r0 = (q0 + q1)/2, and so on. The actual calculations will be with coordinates, not points, but it helps to think in terms of points, keeping in mind that the "average" of two points is the point midway between the two. It turns out that s0 is the midpoint of the Bezier

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curve. What's more, (p0, q0, r0, s0) are (ctrl, hand, hand, ctrl) for a Bezier coinciding with the left half of the original Bezier; and (p3, q2, r1, s0) are (ctrl, hand, hand, ctrl) for another Bezier coinciding with the right half of the original Bezier.

The procedure can be repeated: the midpoint of the left sub-Bezier is 1/4 of the way along the original curve; and the midpoint of the right sub-Bezier is 3/4 of the way along the original curve. The subdivision process can be repeated indefinitely to generate 2, 4, 8, $16,...2^n$ sub-Beziers and 3, 5, 9, $17,...2^n+1$ points along the original curve. Like other subdivision processes, the de Casteljau algorithm lends itself to recursive implementation. Averaging entails dividing by 2, which can be done quickly as a shift right operation because we are dealing with integers.

The de Casteljau construction is illustrated in Figure 2, where the p's are the controls and handles of the original Bezier curve. q0 is midway between p0 and p1, r0 is midway between q0 and q1, and so on. Observe that :

- 1. s0 is the midpoint of the original Bezier curve.
- 2. (p0, q0, r0, s0) are (ctrl1, hand1, hand2, ctrl2) of a sub-Bezier coinciding with the left half of the

original Bezier curve.

3. (p3, q2, r1, s0) are (ctrl1, hand1, hand2, ctrl2) of a sub-Bezier coinciding with the right half of the original Bezier curve.

After the first step illustrated here, we have 3 points on the curve :

The original control point, p0 The midpoint of the original Bezier curve The original control point, p3

Applying the procedure again to the left and right sub-Beziers generates their midpoints, giving five points on the original Bezier. Subdividing these four sub-Beziers then gives us nine points on the original curve, and so on. Stopping at four subdivision levels and 17 points produces smooth curves at VGA resolution. The number of subdivisions, or recursive depth, is BEZ_DEPTH in the program. NUM BEZPTS is the number of points generated along the Bezier curve, and is used to allocate an array to hold the Bezier points. Therefore, make sure that:

NUM_BEZPTS >= 28EZ DEPTR + 1

Increasing BEZ_DEPTH results in more line segments in the Bezier curve, hence a smoother curve—at least up to a point. We reach diminishing returns in increasing BEZ_DEPTH too much, since the accumulated error of repeated averaging eventually throws the calculations off by one or more pixels. Remember that BEZ_DEPTH is an exponent, so increasing BEZ_DEPTH by 1 doubles the number of segments.

Since the recursion proceeds to the maximum depth down the far left branch, the first curve point actually generated is the point immediately following p0 = ctrl1 along the Bezier. The remaining points are also generated in order (from p0 = ctrl1 to p3 = ctrl2), a nice side effect of the recursive implementation. Another point is collected into an array every time the recursion reaches its finest subdivision level, and the points are in order!

Writing tools in Windows—We want to show off our fast Bezier-drawing through an interactive Bezier Tool. If the curve rubber-bands well on the screen, then we can claim to have a good algorithm. Interactive tools in Windows are constructed with the concept of "system state." The **Window** procedure passes mouse messages to **BezTool()**, which maintains a key static variable, **iState**, which takes four values summarized in Table 1.

BezTool()'s action depends on iState, which in turn depends on the sequence of mouse messages that have recently streamed into the tool. Until the first WM LBUTTONDOWN message is received, iState remains NOT STARTED because nothing has been done. The first WM_LBUTTON-DOWN triggers a state transition to DRAG_HAND1. In this state, the tool responds to WM_MOUSEMOVEs by rubber-banding the first handle in XOR mode. WM_LBUTTONUP then causes a state transition to WAIT -FOR CTRL2. Nothing happens until another WM LBUTTONDOWN is received, which changes iState to DRAG HAND2; in this state, WM_MOUSEMOVE messages cause rubber-banding of both the second handle and the Bezier curve as a whole. WM LBUTTONUP now causes a final state transition back to NOT_STARTED. The final handle and Bezier are frozen, and the tool is again ready to start another Bezier.

BezTool() calls DrawHandle() and DrawBez() to draw the figures (which in turn call Windows' GDI calls MoveTo(), LineTo(), and Polyline()). Each mouse move causes two calls to these routines. The first call draws over the figure exactly where it had been drawn the first time. Since we are in XOR drawing mode, drawing over the original figure erases it. The second call then draws at the new location. Static variables must be used if the user points are to be remembered for the next pass through the tool so previous figures can be erased.

Windows and graphics program-

ming—Windows is a natural medium for this kind of programming. Mouse events are sent to our window procedure automatically, enabling us to build interactive mouse-driven tools. The GDI system provides line drawing, including the R2_NOTXORPEN ROP code which allows us to draw in XOR mode.

Windows has a built-in coordinate system and mapping modes so we can change our working range of numbers. The Beziers would not display nearly as nicely were we restricted to screen coordinates of about 500x500 pixels. By setting the **MM_ISOTROPIC** mapping mode and adjusting the Window Extent and Viewport Extent in **BezTool**(), we can expand the range to [-15,000, +15,000] which minimizes the negative side effects of calculations with small integers.

Editor's note: To rebuild BEZ.EXE, you'll need several files in addition to BEZ.C, including some with no ASCII representation. These files are present in a file called BEZ.ZIP, contained within the listings archive for this issue, either on a Disk Subscription disk or from one of the online services and BBS systems that carry our listings.

References

Foley, James D., Andries van Dam, Steven K. Feiner, and John F. Hughes, *Computer Graphics : Principles and Practice*, Addison-Wesley (2nd ed., 1990), pp 507ff.

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struct { int $\alpha[3]$, b; } w[] = { { 1, 2, 3 }, 2 };

Do you see any problems with this declaration? Chances are your compiler will not report any difficulties and yet, you may be surprised to learn how many elements are in w[] (hint: it's not 1). If you need help, give us a call; refer to bug #651.

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▲ TABLE 1—IState's Four Possible Values
                                                                                                CW USEDEFAULT, NULL, NULL, hInstance, NULL);
                                                                          ShowWindow(hWnd, nCmdShow); 7* display the window */
                  ; tool has not been started
NOT STARTED
                                                                          UpdateWindow(hWnd):
                                                                                                    /* update client area; send WM_PAINT */
DRAG HANDI
                  · dragging handlel
WAIT FOR CTRL2
                  : waiting for control2 to be entered
                                                                          /* Read msgs from app que and dispatch them to appropriate win
                  : dragging handle2 and Bezier
DRAG HAND2
                                                                            function. Continues until GetMessage() returns NULL when it
                                                                             receives WM_QUIT. */
▲ LISTING 1—BEZ.C
                                                                          while (GetMessage(&msg, NULL, NULL, NULL))
# BEZ.C : Program to draw Bezier curves and their handles
                                                                            TranslateMessage(&msg); /* process char input from keyboard */
   interactively. User draws first handle by dragging, then second
                                                                            DispatchMessage(&msg); /* pass message to window function */
   handle; the Bezier curve rubber-bands together with the second
   handle. Demonstrates the de Casteljau algorithm for fast
                                                                          return(msg.wParam);
   calculation of Bezier points.
                                                                        }
   Copyright (c) 1991, Michael A. Bertrand. */
                                                                        long FAR PASCAL WndProc(HWND hWnd, unsigned iMessage,
#include <windows.b>
                                                                                               WORD wParam, LONG 1Param)
#include "bez.h'
                                                                          USE: Application's window procedure : all app's messages come
HPEN bRedPen:
                         /* red pen for handles. */
                                                                               here.
int
     LogPerDevice:
                         /* #logical units per device unit
                                                                          IN: hWnd, iMessage, wParam, iParam : standard Windows proc parameters
                            (both axes). */
                                                                        */
                          /* size of client area (x). */
WORD cxClient:
WORD cyClient;
                         /* size of client area (y). */
                                                                        £
                                                                          HDC
                                                                                     hDC; /* must generate our own handle to DC to draw */
HANDLE hInst:
                         /* current instance */
                                                                          PAINTSTRUCT ps;
                                                                                               /* needed when receive WM_PAINT message */
POINT BezPts[NUM_BEZPTS]; /* array of pts along Bezier curve */
                                                                                                      /* pointer to "AboutBez" function */
                                                                          FARPROC 1pProcAbout;
POINT *Ptr8ezPts:
                         /* pointer into BezPts[] array */
                                                                          switch(iMessage)
char Instri[] -
  "* Left Button down, drag, button up for 1st handle.";
                                                                            case WM CREATE:
char Instr2[] -
                                                                              /* Create hRedPen once and store as global. */
  "* Left Button down, drag, button up for 2nd handle and Bez.";
                                                                              hRedPen - CreatePen(PS_SOLID, 1, RGB(255, 0, 0));
char Instr3[] = "* Right click to clear window.":
                                                                              break; /* WM_CREATE */
int PASCAL WinMain(HANDLE hInstance, HANDLE hPrevInstance,
                                                                            case WM_SIZE:
           LPSTR lpszCmdLine, int nCmdShow)
1*
                                                                              /* Get client area size into globals when window resized. */
                                                                              cxCljent = LOWORD(lParam);
 USE: Register window and set dispach message loop.
                                                                              cyClient - HIWORD(1Param);
      hInstance, hPrevInstance, lpszCmdLine, nCmdShow : standard
  IN:
                                                                              break; /* WM_SIZE */
        WinMain parms
*/
                                                                            CASE WM COMMAND:
£
                                                                              if (wParam - IDM ABOUT)
  static char szAppName [] = "Bezier";
  static char szIconName[] - "BezIcon";
                                                                                /* "About" menu item chosen by user :
  static char szMenuName[] = "BezMenu";
                                                                                    call "AboutBez" function. */
                                                                                lpProcAbout = MakeProcInstance(AboutBez, hInst);
  HWND
          hWnd; /* handle to WinMain's window */
                                                                                DialogBox (hInst, "AboutBez", hWnd, 1pProcAbout);
          msg; /* message dispached to window */
  MSG
                                                                                FreeProcInstance(lpProcAbout);
                  /* for registering window */
  WNDCLASS wc:
                                                                                3
                                                                              break; 7* WM_COMMAND */
  /* Save instance handle in global var
     so can use for "About" dialog box. */
                                                                            case WM PAINT:
  hInst = hInstance:
                                                                              /* Repaint instructions at upper left of window. */
                                                                              hDC = BeginPaint(hWnd, &ps);
  /* Register application window class. */
                                                                              SelectObject(hDC, GetStockObject(ANSI_VAR_FONT));
  if (thPrevInstance)
                                                                              TextOut(hDC, 0, 0, Instr1, 1str1en(Instr1));
    {
                                                                              TextOut(hDC, 0, 15, Instr2, 1strlen(Instr2));
    wc.style
                    = CS_HREDRAW | CS_VREDRAW;
                                                                              TextOut(hDC, 0, 30, Instr3, 1strlen(Instr3));
    wc.lpfnWndProc = WndProc; /* fn to get window's messages */
                                                                              EndPaint(hWnd, &ps);
    wc.cbClsExtra = 0;
                                                                              break: /* WM PAINT */
                   = 0;
    wc.cbWndExtra
                    - hInstance;
    wc.hInstance
                                                                            case WM_LBUTTONDOWN:
                 = LoadIcon(hInstance, szIconName);
    wc.hIcon
                                                                            case WM_RBUTTONDOWN:
    wc.hCursor
                   = LoadCursor(NULL, IDC ARROW);
                                                                            case WM_MOUSEMOVE:
    wc.hbrBackground = GetStockObject(WHITE_BRUSH);
                                                                            CASE WM IBUTTONUP:
    wc.lpszMenuName = szMenuName; /* menu resource in RC file */
    wc.lpszClassName = szAppName; /* name used in call to
                                                                              /* Mouse events passed on to BezTool() for processing. */
                                                                              BezTool(hWnd, iMessage, 1Param):
                                  CreateWindow() */
                                                                              break: /* WM_LBUTTONDOWN ... */
    if (!RegisterClass(&wc))
                                                                            case WM_DESTROY:
      return(FALSE):
                                                                              /* Destroy window & delete pen when application terminated. */
    3
                                                                              DeleteObject(hRedPen):
                                                                              PostQuitMessage(0);
    /* Initialize specific instance. */
                                                                              break: /* WM_DESTROY */
    hWnd = CreateWindow(szAppName, szAppName, WS_OVERLAPPEDWINDOW,
                                                                            default:
                        CW_USEDEFAULT, CW_USEDEFAULT, CW_USEDEFAULT,
```

return(DefWindowProc(hWnd, iMessage, wParam, 1Param));) /* switch(iMessage) */ return(OL): void NEAR PASCAL BezTool(HWND hWnd, unsigned iMessage, LONG 1Param) USE: Process mouse event to draw handles and Bezier curve. IN: hWnd : handle to window iMessage : mouse event (WM_LBUTTONDOWN, etc.) 1Param : mouse coords (x --- loword, y --- hiword) NOTE: This is the interactive Bezier drawing tool which processes WM_RBUTTONDOWN, WM_LBUTTONDOWN, WM_MOUSEMOVE, and WM_LBUTTONUP messages. BezTool() is called repeatedly as the user draws. The current state of the tool is maintained in the key static variable iState. iState's value, as set last time thru the tool, determines the tool's action this time thru. Bezier control and handle points, as input by the user, are also maintained as statics so BezTool() remembers them the next time thru. */ { HDC /* must generate our own handle to DC to draw */ hDC: WORD maxClient; /* larger of (cxClient, cyClient) */ /* incoming point */ POINT inPt: POINT pts[2]: /* to get LogPerDevice, #logical units/dev. unit */ /* user-entered Bez control & handle (1st): */ static POINT ctrl1, handl; /* user-entered Bez control & handle (2nd): */ static POINT ctr12, hand2; static int iState; /* BezTool()'s state : DRAG_HAND1, etc. */ hDC - GetDC(hWnd): /* Set extents and origin so will be working in range [-15000, +15000]. */ SetMapMode(hDC, MM_ISOTROPIC); SetWindowExt(hDC, 30000, 30000); maxClient = (cxClient > cyClient) ? cxClient : cyClient; SetViewportExt(hDC, maxClient, -maxClient); SetViewportOrg(hDC, cxClient >> 1, cyClient >> 1); /* Calculate #logical units per device unit -will need later when draw little 3x3 boxes in DrawHandle(). */ pts[0].x = pts[0].y = 0; pts[1].x - pts[1].y - 1; DPtoLP(hDC, pts, 2); LogPerDevice = (pts[1].x > pts[0].x)? (pts[1].x - pts[0].x): (pts[0].x - pts[1].x); /* Incoming point in device coordinates. */ inPt.x = LOWORD(1Param); inPt.y - HIWORD(1Param); /* Convert to logical coordinates. */ DPtoLP(hDC, &inPt; 1); switch(iMessage) £ . case WM RBUTTONDOWN: /* Erase client area if not in middle of Bez. */ if (iState - NOT_STARTED) InvalidateRect(hWnd, NULL, TRUE); break: /* WM_RBUTTONDOWN */ case WM IBUTTONDOWN: switch(iState) 1 case NOT_STARTED: iState = DRAG_HAND1; /* starting drag */ handl.x = ctrll.x = inPt.x; /* store user point handl.y - ctrll.y = inPt.y; in statics */ break; /* NOT_STARTED */ case WAIT_FOR_CTRL2: iState - DRAG_HAND2; /* starting drag */ hand2.x = ctrl2.x = inPt.x; /* store user point hand2.y = ctrl2.y = inPt.y; in statics */

```
SetROP2(hDC, R2_NOTXORPEN); /* draw in XOR */
         DrawBez(hDC, ctrll, handl, hand2, ctrl2);
         break; /* NOT_STARTED */
   /* switch(iState) */
     break; /* WM_LBUTTONDOWN */
   case WM_MOUSEMOVE:
     switch(iState)
       1
       case DRAG_HAND1:
         SetROP2(hDC, R2_NOTXORPEN);
                                        /* draw in XOR */
         DrawHandle(hDC, ctrll, handl); /* erase old */
         handl.x = inPt.x:
                                        /* get new handle */
         handl.y - inPt.y:
         DrawHandle(hDC, ctrll, handl); /* draw new */
         break; /* DRAG_HAND1 */
       case DRAG_HAND2:
         SetROP2(hDC, R2_NOTXORPEN);
                                        /* draw in XOR */
         DrawHandle(hDC, ctrl2, hand2); /* erase old */
         DrawBez(hDC, ctrl1, handl, hand2, ctrl2);
         hand2.x = inPt.x;
                                        /* get new handle */
         hand2.y = inPt.y;
         DrawHandle(hDC, ctrl2, hand2); /* draw new */
         DrawBez(hDC, ctrl1, hand1, hand2, ctrl2);
         break: /* DRAG_HAND1 */
       } /* switch(iState) */
     break; /* WM_MOUSEMOVE */
    case WM_LBUTTONUP:
     switch(iState)
       Ŧ
        case DRAG HAND1:
         iState - WAIT FOR CIRL2:
         SetROP2(hOC, R2_COPYPEN); /* COPY pen for final handle */
         DrawHandle(hOC, ctrl1, handl); /* draw in COPY mode */
         break; /* DRAG_HAND1 */
       case DRAG_HAND2:
         iState = NOT_STARTED;
         SetROP2(hDC, R2_COPYPEN); /* COPY pen for final handle */
         DrawHandle(hDC, ctrl2, hand2); /* draw in COPY mode */
         DrawBez(hDC, ctrl1, handl, hand2, ctrl2);
         break; /* DRAG_HAND2 */
       } /* switch(iState) */
     break; /* WM_LBUTTONUP */
   } /* switch(iMessage) */
  ReleaseDC(hWnd, hDC):
}
BOOL FAR PASCAL AboutBez(HWND hDlg, unsigned iMessage,
                        WORD wParam, LONG 1Param)
USE: Application's "About" dialog box function.
IN: hDlg : handle to dialog box
     iMessage : message type
     wParam : auxiliary message info (act on IDOK, IDCANCEL)
     1Param : unused
RET: Return TRUE if processed appropriate message, FALSE otherwise.
NOTE: Closes "About" box only when user clicks OK button
     or system close. */
ſ
 switch (iMessage)
    case WM_INITDIALOG:
                            /* initialize dialog box */
    return (TRUE):
                             /* received a command */
    case WM COMMAND:
/* IDOK if OK box selected; IDCANCEL if system menu close command */
     if (wParam -- IDOK || wParam -- IDCANCEL)
       EndDialog(hDlg, TRUE); /* exit dialog box */
       return(TRUE);
                               /* did proccess message */
     break: /* WM_COMMAND */
    } /* switch (iMessage) */
  return (FALSE);
                               /* did not process message */
```

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